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Common fixed-point results for nonlinear contractions in ordered partial metric spaces

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Abstract

In this paper, a new class of a pair of generalized nonlinear contractions on partially ordered partial metric spaces is introduced, and some coincidence and common fixed-point theorems for these contractions are proved. Presented theorems are twofold generalizations of very recent fixed-point theorems of Altun and Erduran (Fixed Point Theory Appl 2011(Article ID 508730):10, 2011), Altun et al. (Topol Appl 157(18):2778-2785, 2010), Matthews (Proceedings of the 8th summer conference on general topology and applications, New York Academy of Sciences, New York, pp. 183-197, 1994) and many other known corresponding theorems.
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1 Introduction

It is well known that the Banach contraction principle is a very useful, simple and classical tool in nonlinear analysis. There exist a vast literature concerning its various generalizations and extensions (see [1-45]). In [22], Matthews extended the Banach contraction mapping theorem to the partial metric context for applications in program verification. After that, fixed-point results in partial metric spaces have been studied [4,8,28,31,34,45]. The existence of several connections between partial metrics and topological aspects of domain theory has been pointed by many authors (see [8,9,16,23,31,33,36-38,41,42,46,47]).

First, we recall some definitions of partial metric spaces and some their properties.

Definition 1.1 *A partial metric on a set X is a function $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$:*

(p1) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$,

(p2) $p(x, x) \leq p(x, y)$,

(p3) $p(x, y) = p(y, x)$,

(p4) $p(x, y) \leq p(x, z) + p(z, y) - p(z, z)$.

Note that the self-distance of any point need not be zero, hence the idea of generalizing metrics so that a metric on a non-empty set X is precisely a partial metric p on X such that for any $x \in X$, $p(x, x) = 0$.

Similar to the case of metric space, a partial metric space is a pair (X, p) consisting of a non-empty set X and a partial metric p on X.